Question
State the order and degree of the following differential equations, identify their type and hence solve them.

(i) $\dfrac{dy}{dx} = xy$, where $y = 3$ when $x = 0$;

(ii) $\dfrac{dy}{dx} - 2xy = \dfrac{e^{x^2}}{1+x}$, where $y = 0$ when $x = 0$.

Answer
Both are first order first degree.

(i) This is V.S.

$$\begin{aligned} \frac{dy}{dx} &= xy \\ \Rightarrow \quad \int \frac{dy}{y} &= \int x\,dx \\ \Rightarrow \quad \ln y &= \frac{x^2}{2} + C \\ \Rightarrow \quad y &= Ae^{\frac{x^2}{2}} \\ y = 3 \text{ when x} = 0 & \\ \Rightarrow \quad 3 &= Ae^0 = A \end{aligned}$$

Therefore $\underline{y = 3e^{\frac{x^2}{2}}}$

(ii) This is <u>linear</u>

$$\frac{dy}{dx} - 2xy = \frac{e^{x^2}}{1+x}$$

Integrating factor $= e^{\int -2x\,dx} = e^{-x^2}$

Therefore $e^{-x^2}\dfrac{dy}{dx} - 2xe^{-x^2}y = e^{-x^2}\dfrac{e^{-x^2}}{1+x}$

$\Rightarrow \dfrac{d}{dx}\left\{ye^{-x^2}\right\} = \dfrac{1}{1+x}$

Therefore

$$\begin{aligned} ye^{-x^2} &= \int \frac{dx}{1+x} \\ &= \log(1+x) + c \end{aligned}$$

Therefore $y = e^{x^2}[\log(1+x) + c]$

$y = 0$ when $x = 0$

$$\Rightarrow \begin{array}{rcl} 0 &=& e^0[\log 1 + c] \\ 0 &=& c \end{array}$$

Therefore $\underline{y = e^{x^2}\log(1+x)}$