Question
Calculate to 4 decimal places of accuracy

$$J = \int_0^{\frac{3}{2}} \frac{e^{-x}\cos x}{1+x}\,dx$$

by using

(i) the trapezium rule with 5 ordinates;

(ii) Simpson's rule with 5 ordinates.

(iii) Compare your answers with the exact result $J = 0.439822747...$, calculating the percentage error in each case.

Answer

$$J = \int_0^{\frac{3}{2}} \frac{e^{-x}\cos x}{1+x}\,dx$$

(i) Trapezium rule with 5 ordinates:

$$J \approx \frac{d}{2}(y_1 + 2y_2 + 2y_3 + 2y_4 + y_5)$$

where $d = \dfrac{\frac{3}{2}}{5-1} = \dfrac{3}{8} = 0.375$

$$\begin{array}{ll} x_1 = 0 & x_4 = \frac{9}{8} = 1.125 \\ x_2 = \frac{3}{8} = 0.375 & x_5 = \frac{3}{2} = 1.5 \\ x_3 = \frac{3}{4} = 0.75 & \end{array}$$

$$y_i = f(x_i); \quad f(x) = \frac{e^{-x}\cos x}{(1+x)}$$

x	0	0.375	0.75	1.125	1.5
y	1.00000	0.46511	0.19750	0.06587	0.00631

$$\begin{aligned} J &\approx \frac{0.375}{2}(1.00000 \\ &\quad +2\times(0.46511 + 0.19750 + 0.06587) + 0.00631) \\ &= \frac{0.375}{2}(1.00631 + 1.45697) \end{aligned}$$

$$
\begin{aligned}
&= \frac{0.375}{2} \times 2.46328 \\
&= 0.461866 \\
&= \underline{0.4619}
\end{aligned}
$$

(ii) Simpson with 5 ordinates

$J \approx \dfrac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + y_5)$

4 equal segments $\Rightarrow h = 0.375$ as above.

So we have the same y_i as above.

Hence

$$
\begin{aligned}
J &\approx \frac{0.375}{2}(1.00000 + 4 \times (0.46511 + 0.06587) \\
&\quad +2 \times 0.19750 + 0.00631) \\
&= 0.125(1.00631 + 2.12394 + 0.39500) \\
&= 0.440656 \\
&= \underline{0.4407}
\end{aligned}
$$

(iii) **(i)** $\Rightarrow \left|\dfrac{exact - approx}{exact}\right| \times 100 = 5.019 = 5.0\%$

(ii) $\Rightarrow \left|\dfrac{exact - approx}{exact}\right| \times 100 = 0.199 = 0.2\%$

So (ii), Simpson is here more accurate.