

Question

Calculate f_{xx} , f_{xy} , f_{yx} , f_{yy} for $f(x, y) = x^2 \cos(x + 2y)$

Answer

$$\begin{aligned}f(x, y) &= x^2 \cos(x + 2y) \\f_x &= \frac{\partial f}{\partial x} = 2x \cos(x + 2y) - x^2 \sin(x + 2y) \\f_y &= \frac{\partial f}{\partial y} = -2x^2 \sin(x + 2y) \\f_{xx} &= \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left[\frac{\partial f}{\partial x} \right] \\&= 2 \cos(x + 2y) - 4x \sin(x + 2y) - x^2 \cos(x + 2y) \\f_{xy} &= \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left[\frac{\partial f}{\partial y} \right] \\&= -4x \sin(x + 2y) - 2x^2 \cos(x + 2y) \\f_{yx} &= \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left[\frac{\partial f}{\partial x} \right] \\&= -4x \sin(x + 2y) - 2x^2 \cos(x + 2y) = f_{xy} \\f_{yy} &= \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left[\frac{\partial f}{\partial y} \right] \\&= -4x^2 \cos(x + 2y)\end{aligned}$$