Question

Find $\dfrac{dz}{dt}$, when $z = (x^2 + y^2)^{\frac{1}{2}}$, $x = (t-1)^2$, $y = 2(t-1)$

Answer

$$
\begin{aligned}
\frac{dz}{dt} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}; \qquad \begin{array}{l} x = (t-1)^2 \\ y = 2(t-1) \end{array} \\[2mm]
\frac{dx}{dt} &= 2(t-1); \qquad \frac{dy}{dt} = 2 \\[2mm]
\frac{\partial z}{\partial x} &= \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} \times 2x = \frac{x}{(x^2 + y^2)^{\frac{1}{2}}} \\[2mm]
\frac{\partial z}{\partial y} &= \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} \times 2y = \frac{y}{(x^2 + y^2)^{\frac{1}{2}}} \\[2mm]
\frac{dz}{dt} &= \frac{1}{(x^2 + y^2)^{-\frac{1}{2}}}\{2x(t-1) + 2y\} \\[2mm]
&= \frac{2[(t-1)^3 + 2(t-1)]}{[(t-1)^4 + 4(t-1)^2]^{\frac{1}{2}}} \\[2mm]
&= \frac{2(t-1)([t-1]^2 + 2)}{(t-1)[(t-1)^2 + 4]^{\frac{1}{2}}} \\[2mm]
&= \frac{2[(t-1)^2 + 2]}{[(t-1)^2 + 4]^{\frac{1}{2}}}
\end{aligned}
$$