Question

Consider the element $n(z) = \frac{i\bar{z}+2}{-3\bar{z}+6}$ of the general Möbius group Möb. Determine the set of fixed points of $n(z)$.

Answer

Set $n(z) = z$ and solve for z:

$\dfrac{i\bar{z}+2}{-3\bar{z}+6} = z \Rightarrow -3z\bar{z} + 6z = -\bar{z} + 2$

Set $z = x + iy$ and simplify:

$-3(x^2 + y^2) + 6x + 6iy - i(x - iy) - 2 = 0$

$-3(x^2 + y^2) + 6x - y - 2 + 6iy - ix = 0$

Take imaginary parts: $6y - x = 0 \Rightarrow x = 6y$ and substitute into the equation for the real parts:

$-3(36y^2 + y^2) + 6 \cdot 6y - y - 2 = 0$

$-111y^2 + 35y - 2 = 0$

$$\begin{aligned} y &= \frac{-1}{222}(-35 \pm \sqrt{(35)^2 - 4(-111)(-2)}) \\ &= \frac{-1}{222}(-35 \pm \sqrt{337}) = y_1 \text{ and } y_2 \end{aligned}$$

Two points: $6y_1 + iy_1$ and $6y_1 + iy_2$ are the fixed points of n.