QUESTION

If Z is a normal $N(0,1)$, then is the process $X_t = X\sqrt{t}$ continuous? What is it's distribution? Is X_t a Brownian motion?

ANSWER

$Z \in N(0,1)$ is a continuous random variable. t is continuous time. Thus $Z\sqrt{t}$ is a continuous random variable. It's distribution follows from the standard tranformation between normal distributions as per handout.

If $Z \in N(0,1)$ then

$$X_t = \underbrace{\sqrt{(t)}}_{\text{new standard deviation}} Z + \underbrace{0}_{\text{new mean}}$$

a continuous random variable $X_t \in N(0,t)$. Is it Brownian? Check conditions in notes (p.24)

(i) X_t is continuous and $0 = X_0$

(ii) $X_t \in N(0,t)$

(iii) $X_{s+t} - X_s \in N(0,t)$ and independent of time$<s$: $X_{s+t} - X_s = \left(\sqrt{s+t} - \sqrt{s}\right) Z \in N(0, \sqrt{s+t} - \sqrt{s})$ by above. Therefore (iii) is not satisfied so it is not Brownian.