Ordinary Differential Equations
Classification

Question
Show that $y = -e$ is a solution of $y'' - y = e$.
Find a solution y to satisfy $y(1) = 0$ and $y'(1) = 1$.
Answer
If $y = y_1(x) = -e$ then this will give $y_1' = 0$ and $y_1'' = 0$. Thus

$$y_1'' - y_1 = 0 + e = e.$$

$y_2 = Ae^x + Be^{-x}$ is a solution of $y'' - y = 0$ and so

$$y = y_1(x) + y_2(x) = -e + Ae^x + Be^{-x}$$

is also a solution.
The solution will satisfy

$$\begin{aligned} 0 &= y(1) = Ae + \frac{B}{e} - e \\ 1 &= y'(1) = Ae - \frac{B}{e} \end{aligned}$$

if A and B take the values

$$\begin{aligned} A &= (e+1)/(2e) \\ B &= e(e-1)/2 \end{aligned}$$

So the solution is

$$y = -e + \frac{1}{2}(e+1)e^{x-1} + \frac{1}{2}(e-1)e^{1-x}$$