Ordinary Differential Equations
Classification

Question
If one solution of $y'' + k^2y = 0$ is $y_1 = \cos(kx)$, guess and verify a solution y_2 that is not a multiple of y_1.
Find a solution to satisfy $y(\pi/k) = 3$ and $y'(\pi/k) = 3$.
Answer
As $y_1 = \cos(kx)$ is a solution. A sensible guess for y_2 is $y_2 = \sin(kx)$.
Since
$$y_2'' + k^2y_2 = -k^2\sin(kx) + k^2\sin(kx) = 0$$
then y_2 is confirmed as a solution.
The DE is linear and homogeneous, so any function of the form
$$y = Ay_1 + By_2 = A\cos(kx) + B\sin(kx)$$
is also a solution.
To satisfy the given conditions:
$$\begin{aligned} 3 &= y(\pi/k) = A\cos(\pi) + B\sin(\pi) = -A \\ 3 &= y'(\pi/k) = -Ak\sin(\pi) + Bk\cos(\pi) = -Bk \end{aligned}$$
and so
$$\begin{aligned} A &= -3 \\ B &= -3/k \end{aligned}$$
So the solution is
$$y = -3\cos(kx) - \frac{3}{k}\sin(kx)$$