QUESTION Transistors produced by a machine may be perfect, slightly damaged or unusable. 70% of the production are perfect and 20% are slightly damaged. Let X be a variable giving the number of perfect transistors, Y the number of slightly damaged transistors and Z the number of unusable transistors in a random sample of 3 transistors. Copy out and complete the following table giving the joint and marginal distributions of X and Y.

ANSWER P(perfect) $= 0.7 \sim X$
P(slightly damaged $= 0.2 \sim Y$
P(unusable) $= 0.1 \sim Z$

$$
\begin{aligned}
P(X=0,Y=0) &= P(Z=3) = 0.1^3 = 0.001 \\
P(X=1,Y=0) &= P(X=1,Z=2) = 0.7 \times 0.1^2 \times 3 = 0.021 \\
P(X+1,Y=1) &= P(X=1,Y=1,Z=1) = 0.7 \times 0.2 \times 0,1 \times 6 = 0.084
\end{aligned}
$$

X\Y	0	1	2	3	marginal X
0	*0.001	*0.006	0.012	0.008	0.027
1	*0.021	0.084	0.084	0	0.189
2	0.147	0.294	0	0	0.441
3	0.343	0	0	0	0.343
marginal Y	0.512	0.384	0.096	0.008	1

(i) $X \sim B(3, 0.7)$ $E(X) = 2.1$ Var(X)=0.63

(ii)

	0	1	2	3
P(X—y=0)	$\frac{1}{512}$	$\frac{21}{512}$	$\frac{147}{512}$	$\frac{343}{512}$

$E(X|y=0) = \frac{1}{512}(21 \times 1 + 147 \times 2 + 343 \times 3) = \frac{1344}{512} = 2.625$
$E(X^2|y=0) = \frac{1}{512}(21 \times 1^2 + 147 \times 2^2 + 343 \times 3^2) = \frac{3696}{512} = 7.21875$
$\text{Var}(X|y=0) = 7.21875 - (2.625)^2 \approx 0.328.$

(iii) $P(Z > X+Y)$ $Z = 3 - X - Y > X + Y$ in cells of the table marked with a *.Hence $P(Z > X+Y) = 0.028$.