QUESTION The random variable R can take all non-negative integer values and its pgf is G(z). Show that the probability that R is an odd integer is given be $\frac{(1-G(-1))}{2}$.
An unbiased sis-sided die is thrown repeatedly. Show that the pgf for the number of throws to obtain a six is given by $\frac{z}{(6-5z)}$.
Write down the pgf for the number of throws to obtain two sixes. Two players throw the die in turn. The player who throws the second six is the winner. Find the probability that the game is won by the player who makes the first throw.

ANSWER

$$\begin{aligned} G(z) &= p_0 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots \\ G(-1) &= p_0 - p_1 + p_2 - p_3 + p_4 - p_5 + \ldots \\ 1 = G(1) &= p_0 + p_1 + p_2 + p_3 + p_4 + p_5 + \ldots \\ 1 - G(-1) &= 2(p_1 + p_3 + p_5 + \ldots) \end{aligned}$$

Hence probability that R is an odd integer is $\frac{1-G(-1)}{2} p = \frac{1}{6}$ $p(x) = \frac{5}{6}^{x-1}\frac{1}{6}$ $x = 1, 2, \ldots$ (geometric, number of trials)

$G(z) = \sum_{x=1}^{\infty} z^x (\frac{5}{6})^{x-1}\frac{1}{6} = \frac{z}{6}\sum_{x=1}^{\infty}(\frac{5}{6}z)^{x-1}$
$fracz6(1 - \frac{5}{6}z) = \frac{z}{6-5z}$ For two sixes G(z)=$(\frac{z}{6-5z})^2$(neg.bin n=2)

Probability that the first player wins =Probability throw is 3,5,7,...=$\frac{1}{2}(1-(\frac{-1}{6+5})^2) = \frac{60}{121}$