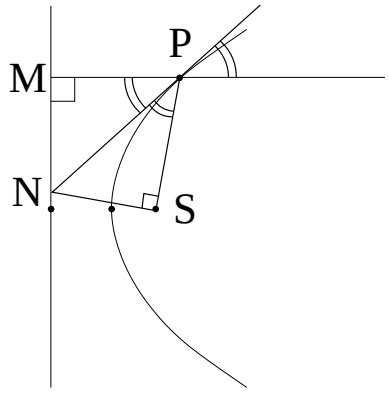
Question

Let P be a point in a parabola with focus S. Let the tangent at P meet the directrix at N. Let M be the foot of the perpendicular from P to the directrix. Show that the angle PSN is a right angle, and use this to deduce the parabolic mirror property

Answer



The equation of the tangent to $x = kt^2$ $y = 2kt$ is $yt = x + kt^2$

This meets the directrix where $x = -k$ so $y = \dfrac{k}{t}(t^2 - 1)$

$S = (k, 0)$ so the gradient of SP is $\dfrac{2kt}{kt^2 - k} = \dfrac{2t}{t^2 - 1}$

The gradient of NS is $\dfrac{\frac{k}{t}(t^2 - 1)}{-k - k} = -\dfrac{t^2 - 1}{2t}$

The product of the gradients is -1 so $N\hat{S}P = 90°$

Now $MP = PS$ by definition of a parabola so the triangles MPS and SPN are congruent (RHS)

Hence the parabolic mirror property is in diagram.