Question

For the curve

$$r = a\cos(\theta - \alpha),$$

show that

$$\phi = \frac{1}{2}\pi + \theta - \alpha$$
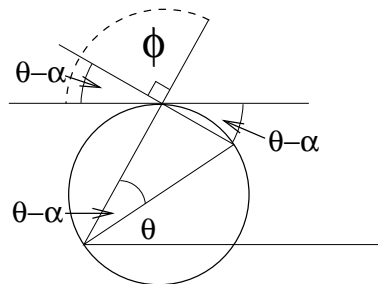
Illustrate this result geometrically.

Answer

$r = a\cos(\theta - \alpha)$

$\cot\phi = \frac{1}{r}\frac{dr}{d\theta} = \frac{-a\sin(\theta-\alpha)}{r} = -\tan(\theta - \alpha)$

So $\phi = \frac{\pi}{2} + \theta - \alpha$

This is the alternate segment theorem.