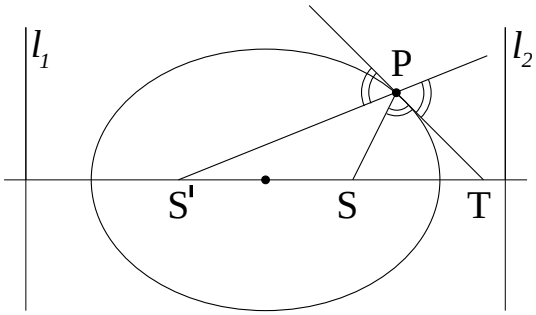
**Question**
An ellipsoid is generated by by rotating an ellipse about its major axis. The inside surface of the ellipsoid id silvered to produce a mirror. Show that a ray of light emanating from one focus will be reflected to the other focus.

**Answer**
**Either**

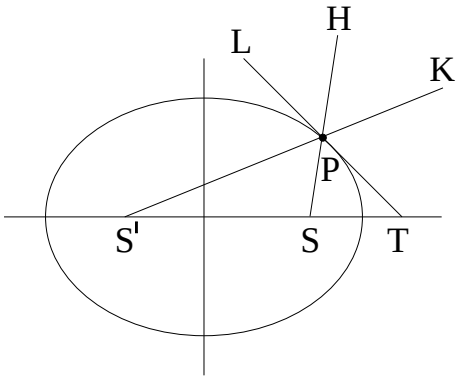



The equation to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(x_0, y_0)$ i s

$$\frac{x_0}{a^2} + \frac{yy_0}{b^2} = 1$$

so $y = 0$ gives $x = \frac{a^2}{x_0}$ which is the x coordinate of T.
So $ST = \frac{a^2}{x_0} - ae \quad S'T = \frac{a^2}{x_0} + ae$
The distance of P from the directrix $l_1$ is $\frac{a}{e} - x_0$
So $SP = e\left(\frac{a}{e} - x_0\right) = a - ex_0. \qquad s'P = a + ex_0$
So $\frac{ST}{s'T} = \frac{SP}{S'P}$
Thus $PT$ is an external bisector of $SPS'$ hence by the angles bisector theorem (converse) the angles are equal as marked.
Hence the reflection property.

**or**




Let P be any point on the ellipse with coordinates $(x_0, y_0)$

$$\begin{aligned}
\text{The gradient of PT is } -\frac{b^2 x_0}{a^2 y_0} &= m_1 \\
\text{The gradient of S}'\text{P is } \frac{y_0}{x_0 + ae} &= m_2' \\
\text{The gradient of SP is } \frac{y_0}{x_0 - ae} &= m_2
\end{aligned}$$

$$\begin{aligned}
\tan K\hat{P}T &= \frac{m_1 - m_2'}{1 + m_1 m_2'} \\
&= \frac{\dfrac{y_0}{x_0 + ae} + \dfrac{b^2 x_0}{a^2 y_0}}{1 - \dfrac{y_0}{x_0 + ae} \cdot \dfrac{b^2}{a^2 y_0}} \\
&= \frac{a_2 y_0^2 + b^2 x_0^2 + b^2 a e x_0}{(a^2 - b^2) x_0 y_0 + a^3 y_0 e} \\
&= \frac{a^2 b^2 + b^2 a e x_0}{a^3 y_0 e + a^2 e^2 x_0 y_0} \\
&= \frac{b^2}{a e y_0}
\end{aligned}$$

To obtain $\dfrac{m_1 - m_2}{1 + m_1 m_2}$ replace $a$ by $-a$ in above, so that $\tan H\hat{P}T = -\dfrac{b}{a y_o e}$
So $K\hat{P}T = \pi - H\hat{P}T$
Therefore $L\hat{P}S' = S\hat{P}T$ QED