

Question

An ellipse has polar equation

$$\frac{2}{r} = 1 - \frac{1}{3} \cos \theta.$$

Find the cartesian equation to axis having their origin at the centre of the ellipse.

Answer

$$\frac{2}{r} = 1 - \frac{1}{3} \cos \theta \quad \text{So } l = 2 \text{ and } e = \frac{1}{3}$$

The cartesian equation is $\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$

When $x = \frac{1}{3}a$ and $y = l (= 2)$ then

$$\frac{1}{9} \frac{a^2}{a^2} + \frac{4}{a^2 \left(\frac{8}{9}\right)} = 1 \quad \text{So } a = \frac{9}{4} \quad b^2 = \frac{9}{2}$$

So the equation is

$$16x^2 + 18y^2 = 81$$