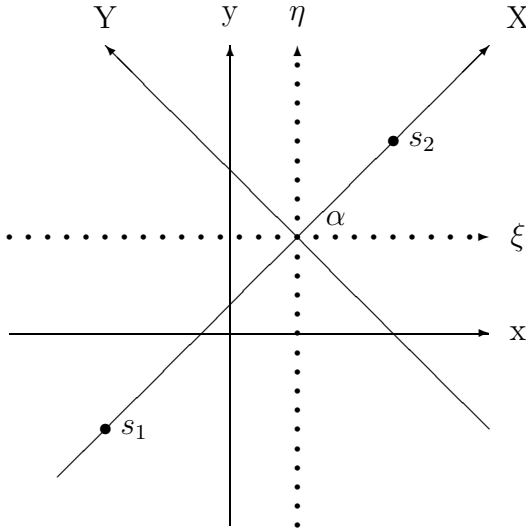


Question

Find the equation of the ellipse with foci $(-1, -1)$, $(2, 3)$ and major axis of length 10. Find the centre of this ellipse and calculate its eccentricity.

Answer



$s_1 = (-1, -1)$
 $s_2 = (2, 3)$
 $s_1 s_2 = 5$
 Major axis = 10
 so $e = \frac{1}{2}$
 The centre is $(\frac{1}{2}, 1)$
 "a" = 5

Referred to the X-Y axis the equation is

$$\frac{x^2}{25} + \frac{Y^2}{25(1 - \frac{1}{4})} = 1 \quad \frac{X^2}{25} + \frac{4y^2}{75} = 1 \Rightarrow 3X^2 + 4Y^2 = 75$$

X-Y axis are obtained from $\xi - \eta$ axis where $\sin \alpha = \frac{4}{5}$ $\cos \alpha = \frac{3}{5}$

$$\begin{aligned}
 X &= \xi \cos \alpha + \eta \sin \alpha = \frac{3}{5}\xi + \frac{4}{5}\eta \\
 Y &= \eta \cos \alpha - \xi \sin \alpha = \frac{3}{5}\eta - \frac{4}{5}\xi
 \end{aligned}$$

which gives

$$91\xi^2 - 24\xi\eta + 84\eta^2 = 1875$$

Now $\xi = x - \frac{1}{2}$ and $\eta = y - 1$

So the equation is

$$364x^2 - 96xy + 336y^2 - 268x - 624y - 7121 = 0$$