Question

(a) Can the torque on a particle be zero without the force being zero? Explain.

(b) Can the force on a particle be zero without the angular momentum being zero? Explain.

Answer

(a) If the torque is zero then $\mathbf{r} \times \mathbf{F} = 0$

Now $\mathbf{N} = \mathbf{r} \times \mathbf{F}$ hence $\mathbf{N} = 0 \Rightarrow \mathbf{r}$ or $\mathbf{F}$ is zero, or $\mathbf{r}$ and $\mathbf{F}$ are parallel.

i.e. yes. either $\mathbf{r} = 0$ (the force is acting at the origin) or $\mathbf{r}$ and $\mathbf{F}$ are parallel; and so $\mathbf{F}$ is acting in a direction through the origin.

(b) $\mathbf{F} = 0 \Rightarrow \mathbf{N} = \mathbf{r} \times \mathbf{F} = 0$

Using Newton's 2nd law: $0 = \mathbf{N} = \dot{\mathbf{L}} \Rightarrow \mathbf{L}$ = constant. Thus $\mathbf{L}$ just needs to be constant (not necessarily zero).

Answer: Yes.