QUESTION

**(a)** Find a unit normal to the surface $xyz + 3x^2 = 4$ at the point (1,1,1).

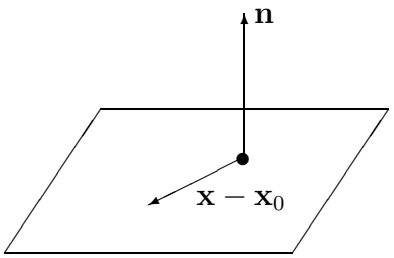
**(b)** Find the equation of the tangent plane to the surface $3yz^2+2x^2-4xy^2 = 3$ at the point $(0, 1, 1)$.

**(c)** Find the angle between the surfaces $2x^2+y^2+z^2 = 4$ and $x^2+y^2-z = 1$ at the point $(1, 1, 1)$.

ANSWER

**(a)** $f = xyz + 3x^2 - 4$ $\mathbf{x}_0 = (1, 1, 1)$
$\nabla f = (yz + 6x, xz, xy)$ $\nabla f|_{\mathbf{x}_0} = (7, 1, 1)$ $\mathbf{n} = \pm\frac{1}{\sqrt{51}}(7, 1, 1)$

**(b)** $f = 3yz^2 + 2x^2 - 4xy^2 - 3$ $\mathbf{x}_0 = (0, 1, 1)$
$\nabla f = (4x - 4y^2, 3z^2 - 4x, 6yz)$ $\nabla f|_{\mathbf{x}_0} = (-4, 3, 6)$
Equation of tangent plane is $(\mathbf{x} - \mathbf{x}_0) \cdot \mathbf{n} = 0$




$-4(x - 0) + 3(y - 1) + 6(z - 1) = 0$
$-4x + 3y + 6z = 9$

**(c)** $f_1 = 2x^2 + y^2 + z^2 - 4$ $f_2 = x^2 + y^2 - z - 1$ $\mathbf{x}_0 = (1, 1, 1)$
$\nabla f_1 = (4x, 2y, 2z)$ $\mathbf{n}_1 = \frac{1}{\sqrt{24}}(4, 2, 2) = \frac{1}{\sqrt{6}}(2, 1, 1)$
$\nabla f_2 = (2x, 2y, -z)$ $\mathbf{n}_2 = \frac{1}{3}(4, 2, 2)$
Angle between surfaces = angle between normals = $\theta$
$\cos\theta = \mathbf{n}_1 \cdot \mathbf{n}_2 = \frac{1}{\sqrt{6}}\frac{1}{3}5$
$\theta = \cos^{-1}\left(\frac{5}{3\sqrt{6}}\right)$