QUESTION

A European call option is to be priced using the binomial model assuming the following data

Strike \$50;
Maturity 1 year, two intervals;
Continuously compounded annual risk free interest at 3%;
Volatility of underlying stick 30%;
Current price \$50

(a) Show that the up and down factors for the share price $U,\ D$ respectively, over a six month period are $U = 1.227073,\ D = 0.802814$. What is the continuously compounded interest rate for each six month period? Calculate the asset prices, representing this information on a binomial tree.

(b) By constructing a replicating portfolio of shares and cash and working to 5 decimal places, calculate the initial premium for the option.

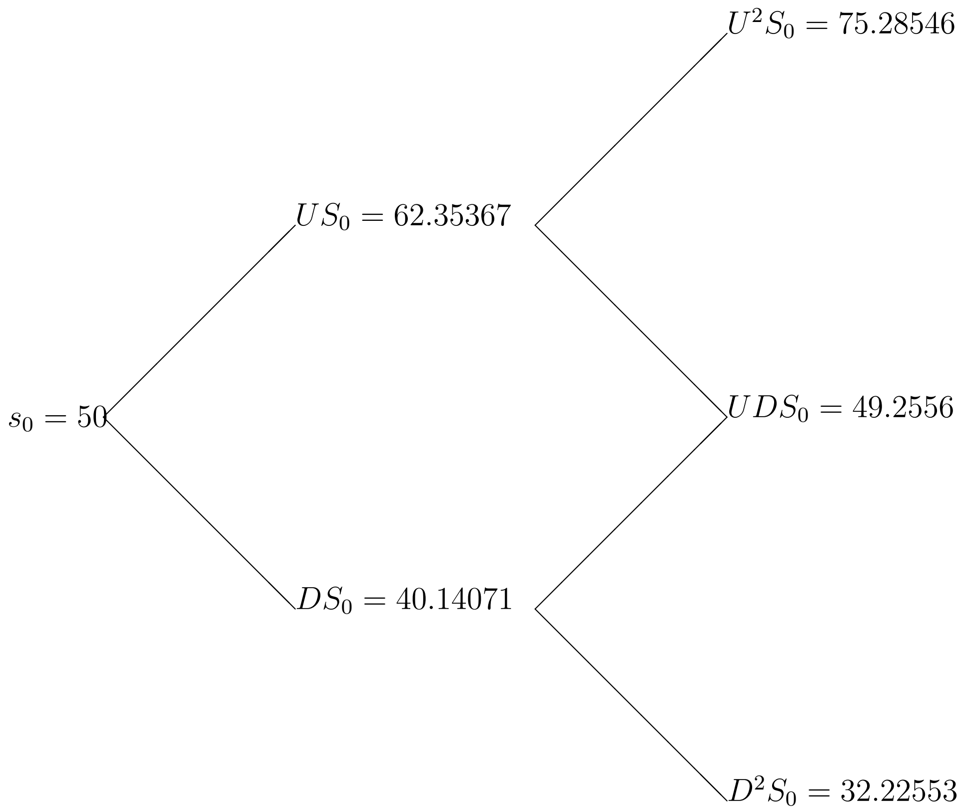
(c) Discuss briefly the trading strategy for the write of the option if the underlying share always rises in value.

ANSWER

$K = 50\, T = 1,\ \delta t = \frac{1}{2},\ r = 3\% = 0.03,\ \sigma = 30\% = 0.3,\ S_0 = 50$

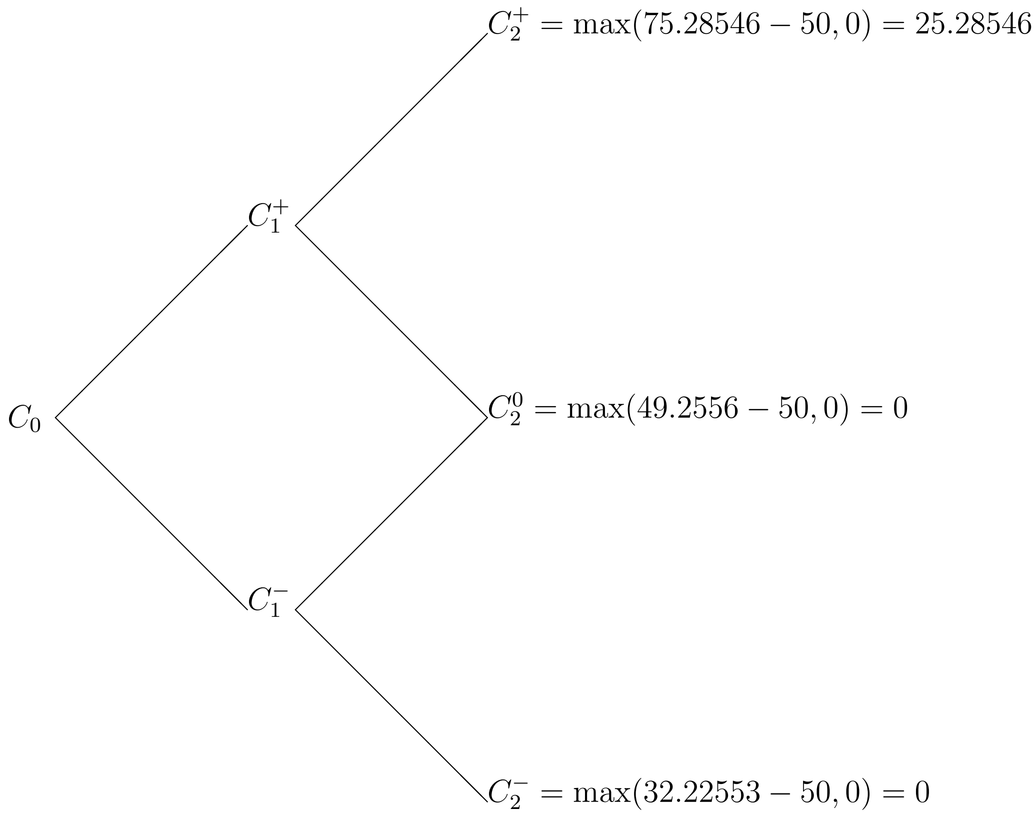
(a) $U = e^{\left(1-\frac{\sigma^2}{2}\right)\delta t+\sigma\sqrt{\delta t}} = 1.227073$

$D = e^{\left(1-\frac{\sigma^2}{2}\right)\delta t-\sigma\sqrt{\delta t}} = 0.802814$



Continuously compounded interest rate= $e^{r\delta t} = e^{0.03\times 0.5} = 1.015113$

(b) Calculate final value of option $= \max\{s - k, o\}$



Calculate $C_1^+,\ C_1^-,\ C_0$ by stepping back from $C_2^+,\ C_2^0,\ C_2^-$

Replicating portfolio matches values of C_m^n

$\underline{t = 1 \text{ UP}}$

Construct a portfolio at $t = 1$ in up state, ψ_1^+ shares, ψ_1^+ cash.

$$C_1^+ = 61.35367\phi_1^+ + \psi_1^+$$

so at $t = 2$, if up-state $(U^2 S_0)$

$C_2^+ = 75.28546\phi_1^+ + 1.015113\psi_1^+ = 25.28546$

and at $t = 2$, if in down-state (UDS_0)

$C_2^- = 49.25556\phi_1^+ + 1.0151134\psi_1^+ = 0$

Therefore

$\phi_1^+ = \frac{25.28546}{(75.28546-49.2556)} = 0.971402$
$\psi_1^+ = -\frac{49.2556}{1.015113} \times 0.971402 = -47.134642$

Therefore

$C_1^+ = 61.35367 \times 0.971402 - 47.134642 = 12.464436$

$\underline{t = 1 \text{ DOWN}}$

Portfolio is ϕ_1^- shares and ψ_1^- cash.

$$C_1^- = 40.14071\phi_1^- + \psi_1^-$$

So at $t = 2$ if in up-state (UDS_0)

$C_2^0 = 49.2556\phi_1^- + 1.015113\psi_1^- = 0$

and at $t = 2$ if in down-state (D^2S_0)

$C_2^- = 32.22553\phi_1^- + 1.015113\psi_1^- = 0$
$\Rightarrow \phi_1^- = \psi_1^- = 0$ (No portfolio needed)

Therefore $C_1^- = 0$.

$\underline{t = 0}$

Portfolio is ϕ_0 shares, ψ_0 cash.

$$C_0 = 50\phi_0 + \psi_0$$

So at $t = 1$, if up-state (US_0)

$C_1^- = 40.14071\phi_0 + 1.015113\psi_0 = 0$

Therefore

$\phi_0 = \frac{12.464436}{(61.35367-40.1407)} = 0.587586$
$\psi_0 = -\frac{40.14071 \times 0.587586}{1.015113} = -23.234969$

Therefore

$C_0 = 50 \times 0.587586 - 23.234969 = 6.144331$ (initial price)

(c)

$t = 0$	buy 0.587586 shares @ 50, financed by	
	23.234969 borrowing	
	and 6.144331 premium	
	Net cost=$(50 \times 0.587586) - 23.234969 - 6.14431 = 0$	
$t = 1$	Buy $(0.971402 - 0.587586)$ shares @ 61.35367	
	financed by borrowing 0.383816×61.35367	
	which takes borrowing to $23.548520 + 1.015113\times$	
	$23.234969 = 47.134642$	
$t = 0$	Buy $(1 - 0.971402)$ shares @75.28546cost=	2.153014
	sell 1 share @ 50 to owner of all recoup=	-50
	Payback borrowing $1.015113 \times 47.134642$cost=	47.846988
	net=	0