

Question

\mathcal{C} is a countable set. Prove that for any set S ,
 $m^*(S \cup \mathcal{C}) = m^*(S)$

Answer

$$m^*(S \cup \mathcal{C}) \leq m^*(S) + m^*(\mathcal{C}) = m^*(S)$$

But $S \subset S \cup \mathcal{C}$ therefore $m^*(S) \leq m^*(S \cup \mathcal{C})$

Therefore $m^*(S) = m^*(S \cup \mathcal{C})$