Question
In $\mathbf{R^n}$, show that if $S = \{(x_1, \cdots x_n) : x_r = a\}$ then $m^*(S) = 0$.

Answer
Let $R_n = \{(x_1, \cdots x_n) : |x_i| \le n, \ \ i \ne r, \ \ a - \epsilon_n \le x_r \le a + \epsilon_n\}$
where $\epsilon_n = \epsilon \left(\frac{1}{2}\right)^{n+1} \frac{1}{(2n)^{n-1}}$
Then $\bigcup_{n=1}^{\infty} R_n \supseteq S$
$|R_n| = (2n)^{n-1} 2\epsilon_n = \frac{\epsilon}{2^n}$ Therefore $\sum_{n=1}^{\infty} |R_n| = \epsilon$
Thus $m^*(S) = 0$