QUESTION

Find the eigenvalues and eigenvectors of A and hence find A^7 where

$$A = \begin{bmatrix} 29 & -15 \\ 50 & -26 \end{bmatrix}.$$

ANSWER

The eigenvalues are 4 and -1 with eigenvectors $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ respectively.

Hence $A^r = M\Lambda^r M^{-1}$ where $\Lambda =$ diag(4,-1), and the columns of M are the eigenvectors, so

$$\begin{aligned}
A^r &= \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 4^r & 0 \\ 0 & (-1)^r \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 6 \times 4^r - 5(-1)^r & -3(4^r - (-1)^r) \\ 10(4^r - (-1)^r) & -5 \times 4^r + 6(-1)^r \end{bmatrix} \\
A^7 &= \begin{bmatrix} 98309 & -49155 \\ 163850 & -81926 \end{bmatrix}
\end{aligned}$$