

Question

1. A ball has **bounce coefficient** $0 < r < 1$ if, when it is dropped from height h , it bounces back to a height of rh . Suppose that such a ball is dropped from the initial height a and subsequently bounces infinitely many times. Determine the total up-and-down distance the ball travels.
2. Two cars, driven by Jack and Jill, are begin driven towards each other, with Jack driving at 25 miles per hour and Jill driving at 95 miles per hour. When the cars are 120 miles apart, a fly leaves the front of Jack's car and flies to Jill's car at 257 miles per hour; when it reaches Jill's car, it immediately turns around and flies back to Jack's car, and keeps going back and forth until it is crushed between the two cars when they crash together. Assuming the fly loses no time in changing direction, calculate the total distance the fly has flown in its journey between the two cars.

Answer

1. Before hitting the ground the first time, the ball travels distance a . Between hitting the ground the first and second times, the ball travels distance $2ra$ (distance ra up from the ground, and then distance ra back to down to earth again). Between hitting the ground the second and third times, the ball travels distance $2r^2a$ (distance r^2a up from the ground, and then distance r^2a back to down to earth again). Between hitting the ground the n^{th} and the $(n + 1)^{\text{st}}$ times, the ball travels distance $2r^n a$ (distance $r^n a$ up from the ground, and then distance $r^n a$ back to down to earth again). Hence, the total distance travelled is

$$\begin{aligned} a + 2ra + 2r^2a + \dots &= a + \sum_{n=1}^{\infty} 2r^n a = a + 2ra \sum_{n=1}^{\infty} r^{n-1} = a + 2ra \sum_{k=0}^{\infty} r^k \\ &= a + \frac{2ra}{1-r} = \frac{a+ra}{1-r}. \end{aligned}$$

2. One way to do this problem is to actually write out the appropriate geometric series and summing it. The easier way is to note that the cars will crash exactly one hour after the fly leaves the front of Jack's car, and in that hour (given the assumption that the fly loses no time in changing direction) the fly flies exactly 257 miles.