Question
Let $\sum_{n=1}^{\infty} a_n$ be a convergent series of positive terms. Show that for each $s \geq 1$, the series $\sum_{n=1}^{\infty} a_n^s$ is also convergent.

Answer
By the contrapositive to the n^{th} term test for divergence, since the series $\sum_{n=1}^{\infty} a_n$ converges, we have that $\lim_{n\to\infty} a_n = 0$. In particular, taking $\varepsilon = 1$ and remembering that each $a_n > 0$, there exists M so that $0 < a_n < 1$ for all $n > M$. Since $0 < a_n < 1$ for $n > M$ and since $s \geq 1$, we have that $a_n^s < a_n$ for $n > M$, and so by the second comparison test, we have that $\sum_{n=M+1}^{\infty} a_n^s$ converges by comparison to $\sum_{n=M+1}^{\infty} a_n$. Since $\sum_{n=M+1}^{\infty} a_n^s$ converges, we see that $\sum_{n=0}^{\infty} a_n^s$ converges, as desired.