

### Exam Question

#### Topic: CriticalPoints

Find and classify the critical points of the function

$$f(x, y) = 3x^4 + 12xy + 4y^3.$$

#### Solution

$$f(x, y) = 3x^4 + 12xy + 4y^3.$$

$$f_x = 12x^3 + 12y = 0 \Rightarrow y = -x^3.$$

$$f_y = 12x + 12y^2 = 0 \Rightarrow x = -y^2.$$

So  $x = -x^6$ , giving  $x = 0$  or  $x^5 = -1$  i.e.  $x = -1$ .

The critical points are therefore  $(0, 0)$ ;  $(-1, 1)$

	$(0, 0)$	$(-1, 1)$
$f_{xx} = 36x^2$	0	36
$f_{xy} = 12$	12	12
$f_{yy} = 24y$	0	24
$\Delta = f_{xy}^2 - f_{xx}f_{yy}$	144	-720

So  $(0, 0)$  is a saddle point, and since  $f_{xx}(-1, 1) > 0$ ,  $(-1, 1)$  is a local minimum.