Vector Calculus
Grad, Div and Curl

Question

Calculate **divF** and **curlF** for the vector field

$\underline{F} = \underline{\hat{\theta}} = -\sin\theta\underline{i} + \cos\theta\underline{j}$

Answer

$$\mathrm{div}\underline{F} = \frac{\cos\theta\sin\theta}{r} - \frac{\cos\theta\sin\theta}{r} = 0$$

$$\mathrm{curl}\underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\sin\theta & 7\cos\theta & 0 \end{vmatrix}$$

$$= \left(\frac{\sin^2\theta}{r} + \frac{\cos^2\theta}{r}\right)\underline{k} = \frac{1}{r}\underline{k}$$

$$= \frac{1}{\sqrt{x^2 + y^2}}\underline{k}$$