

Vector Calculus
Grad, Div and Curl

Question

Calculate $\mathbf{div}\mathbf{F}$ and $\mathbf{curl}\mathbf{F}$ for the vector field

$$\underline{F} = \underline{\hat{r}} = \cos \theta \underline{i} + \sin \theta \underline{j}$$

Answer

$$\begin{aligned} \mathbf{div}\underline{F} &= \frac{\sin^2 \theta}{r} + \frac{\cos^2 \theta}{r} = \frac{1}{r} \\ &= \frac{1}{\sqrt{x^2 + y^2}} \\ \mathbf{curl}\underline{F} &= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos \theta & \sin \theta & 0 \end{vmatrix} \\ &= - \left(\frac{\cos \theta \sin \theta}{r} - \frac{\cos \theta \sin \theta}{r} \right) \underline{k} = \underline{0} \end{aligned}$$