Question

Find the general solution for each of the following second order equations:

(a) $y'' - 3y' + 2y = 0$

(b) $y'' + 4y' + 13y = 0$

(c) $y'' - 4y' + y = 0$

(d) $y'' - 3y' = 0$

Answer

(a)

$$
\begin{aligned}
y'' - 3y' + 2y &= 0 \\
\text{A.E.} \ \ m^2 - 3m + 2 &= 0 \\
(m-1)(m-2) &= 0 \\
m = 1 \ \ \text{or} \ \ m &= 2 \\
\text{so} \ \ y &= Ae^x + Be^{2x}
\end{aligned}
$$

(b)

$$
\begin{aligned}
y'' + 4y' + 13y &= 0 \\
\text{A.E.} \ \ m^2 + 4m + 13 &= 0 \\
(m+2)^2 &= -9 \\
m = -2 \pm 3i & \\
\text{so} \ \ y &= e^{-2x}(A\cos 3x + B\sin 3x)
\end{aligned}
$$

(c)

$$
\begin{aligned}
y'' - 4y' + y &= 0 \\
\text{A.E.} \ \ m^2 - 4m + 1 &= 0 \\
m = 2 \pm \sqrt{3} & \\
\text{so} \ \ y &= e^{2x}(Ae^{\sqrt{3}x} + Be^{-\sqrt{3}x})
\end{aligned}
$$

(d)

$$\begin{aligned} y'' - 3y' &= 0 \\ \text{A.E.} \quad m^2 - 3m &= 0 \\ m(m - 3) &= 0 \\ m = 0 \quad \text{or} \quad m = 3 \\ \text{so} \quad y &= A + Be^{3x} \end{aligned}$$