

QUESTION

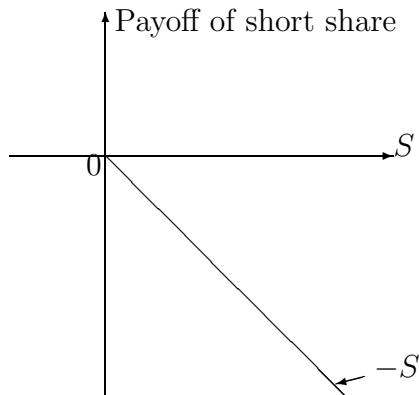
Draw the expiry payoff diagrams for each of the following portfolios (ignore premium costs):

- (a) Short one share, long two calls with exercise price K (this one is called a straddle);
- (b) Long one call and one put, both with exercise price K (this is also a straddle);
- (c) Long one call, and two puts, all with exercise price K (a strap);
- (d) Long one put and two calls, all with exercise price K (a strip);
- (e) Long one call with exercise price K_1 and one put with exercise price K_2 ; compare the three cases $K_1 > K_2$ (also a strangle), $K_1 = K_2$ and $K_1 < K_2$;
- (f) As in (e), but also short one call and one put with exercise price K , where now $K_1 < K < K_2$ (a butterfly spread).

ANSWER

In what follows payoff is drawn for the owner of the portfolio and ignores premium.

- (a) “Short” means you have a liability to but a share rather than being “long” which means you own it. Thus the payoff from a short share is

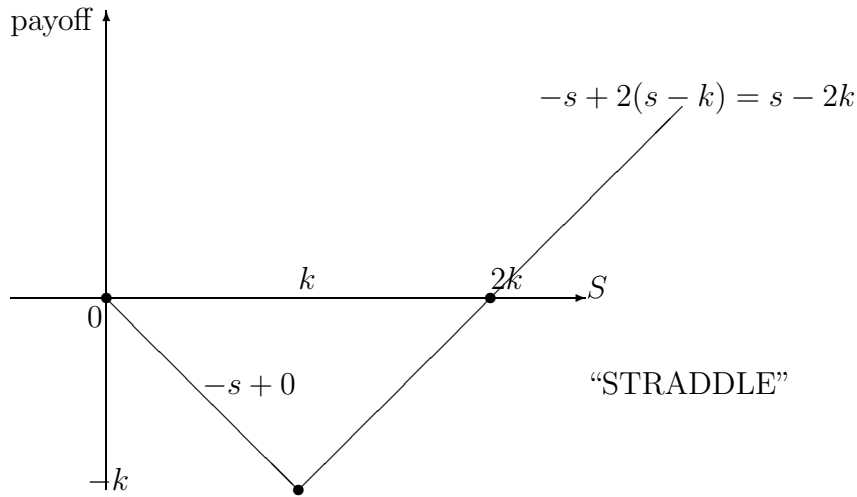


Payoff of a call to the owner is $= \max(s - k, 0)$

Thus payoff of 2 calls is $= 2 \times \max(s - k, 0)$

Thus the total payoff of portfolio is $= -s + 2 \max s - k, 0$

Graphically

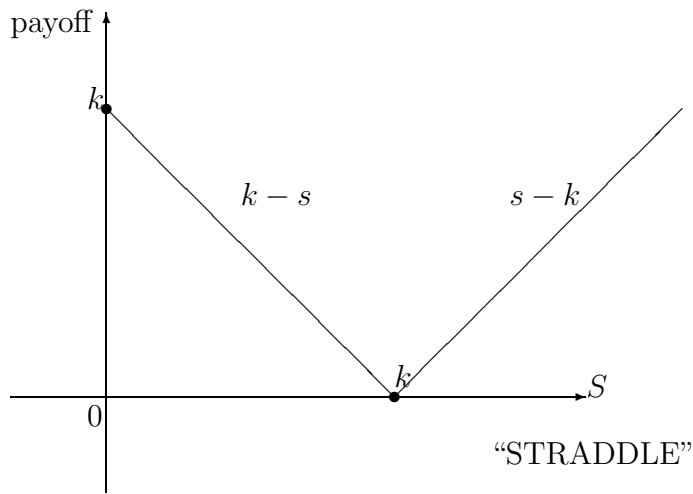


(b) Long call payoff = $\max(s - k, 0)$

Long put payoff = $\max(k - s, 0)$

$$\text{Total payoff} = \max(s - k, 0) + \max(k - s, 0) = \begin{cases} k - s, & k \geq s \\ s - k, & k \leq s \end{cases} \quad \text{Similar}$$

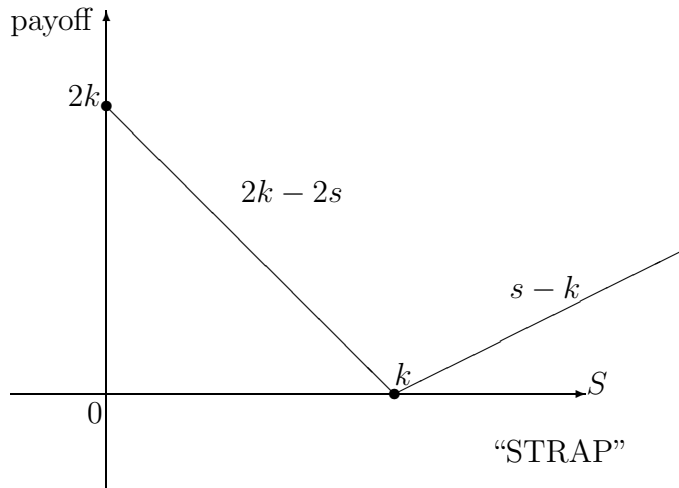
shape to other “straddle” hence name.



(c) Long one call = $\max(s - k, 0)$

Long two puts = $2 \max(k - s, 0)$

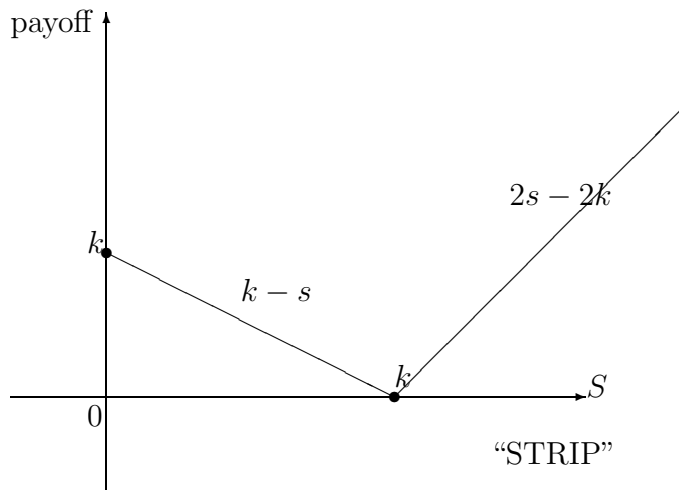
$$\text{Total payoff} = \max(s - k, 0) + 2 \max(k - s, 0) = \begin{cases} s - k, & s \geq k \\ 2(k - s), & s \leq k \end{cases}$$



(d) Long one put = $\max(k - s, 0)$

Long two calls = $2 \max(s - k, 0)$

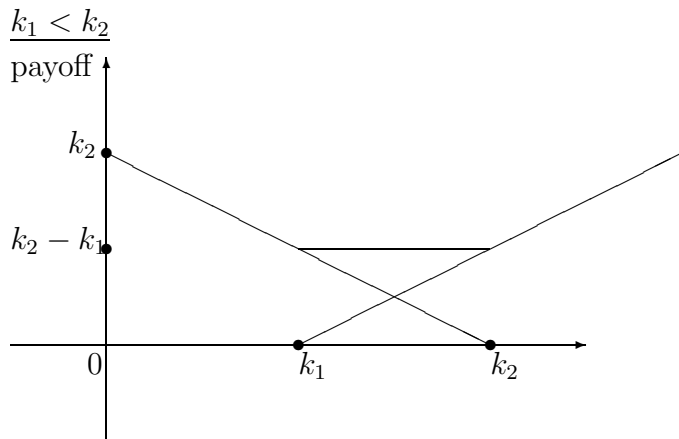
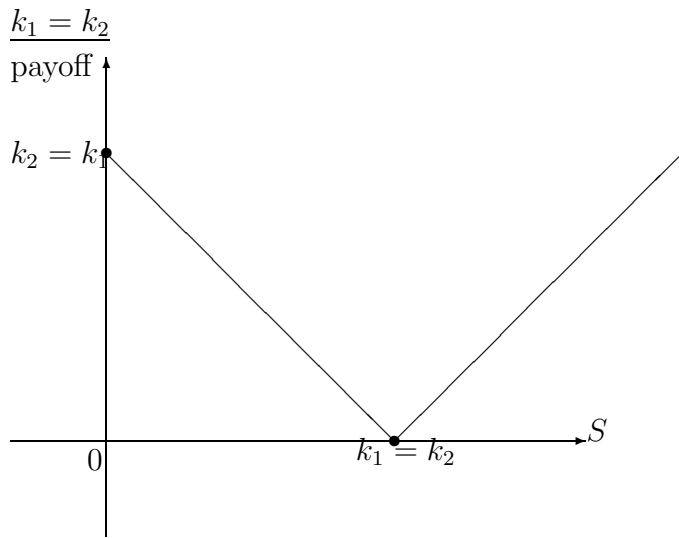
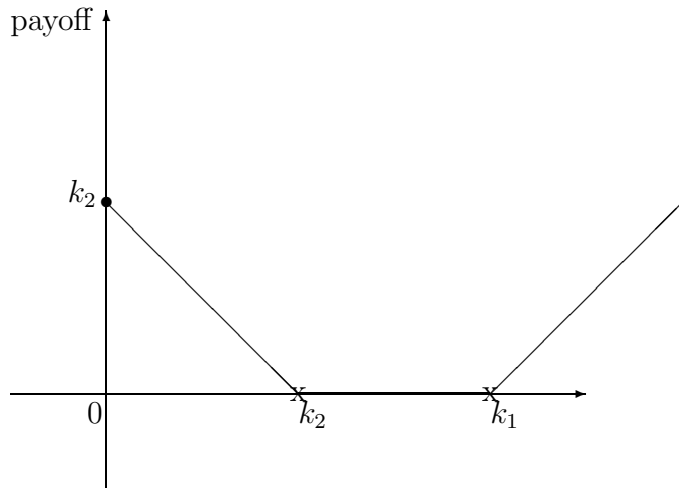
$$\text{Total payoff} = \max(k - s, 0) + 2 \max(s - k, 0) = \begin{cases} 2(s - k), & s \geq k \\ k - s, & s \leq k \end{cases}$$



(e) Long one call, strike $k_1 = \max(s - k_1, 0)$

Long one put, strike $k_2 = \max(k_2 - s, 0)$

$$\underline{k_1 > k_2}$$



(f) Short one call = $-\max(s - k, 0)$

Short one put = $-\max(k - s, 0)$, $k_1 < k < k_2$

Long one call = $\max(s - k, 0)$

Long one put = $\max(k_2 - s, 0)$

Total payoff = $\max(k_2 - s, 0) + \max(s - k_1, 0) - \max(k - s, 0) - \max(s - k, 0)$

Split into several ranges

$$\begin{aligned}
 0 < s < k_1 (< k < k_2) &\Rightarrow & -(k - s) + k_2 - s &= k_2 - k \\
 0 < k_1 < s < k < k_2 &\Rightarrow & -(k - s) + s - k_1 + k_2 - s &= k_2 - k - k_1 + s \\
 0 < k_1 < k < s < k_2 &\Rightarrow & -(s - k) + s - k_1 + k_2 - s &= k_2 + k - k_1 - s \\
 0 < k_1 < k < k_2 < s &\Rightarrow & -(s - k) + s - k_1 &= k - k_1
 \end{aligned}$$

Graphically:

