Question

(∗) It is November 5 and a rocket of mass $50g$ is stationary on the ground pointing vertically to the sky. The fuse if lit and the motor, generating a force of $1{,}500mg/s^2$ (metres grams per (second)^2), burns for two seconds before it stops. (You may wish to take the gravitational constant as $10m/s^2$).

1. Write down the equation for the velocity during the first two seconds of flight (neglect the changing mass of the rocket as fuel is used, neglect air resistance but include gravity).

   Find the velocity and the position for the first two seconds.

2. Find the velocity and position of the rocket for times later than two seconds after liftoff (during this time only the gravitational force acts on the rocket).

   Determine the maximum height of the rocket.

   Calculate the total amount of time the rocket is in the air before it crashes back to the ground.

Answer

$0 \leq t \leq 2$

mass = 50, motor forces = 1500, gravity = 10

$50\dfrac{dv}{dt} = 1500 - 50 * 10, \quad \dfrac{dx}{dt} = v$

$\dfrac{dv}{dt} = 20, \quad \dfrac{dx}{dt} = v, \quad$ at $t = 0, \quad v = 0, \quad x = 0$

$\Rightarrow v = 20t, \quad x = 10t^2$

hence at $t = 2, \quad v = 40m/s, \quad x = 40m$

$2 \leq t$

mass = 50, motor force = 0, gravity = 10

$50\dfrac{dv}{dt} = -50 * 10, \quad \dfrac{dx}{dt} = v$

$\dfrac{dv}{dt} = -10, \quad \dfrac{dx}{dt} = v, \quad$ at $t = 2, \quad v = 40, \quad x = 40$

$\Rightarrow v = -10t + 60, \quad x = -5t^2 + 60t - 60$

The maximum height is when $v = \dfrac{dx}{dt} = 0 \Rightarrow -10t + 60 = 0$

$\Rightarrow t = 6$ seconds

$x(6) = -5(36) + 60(6) - 60 = 120\text{m}$
The rocket returns to the ground when $x = 0 \Rightarrow 0 = -5t^2 + 60t - 60$
$\Rightarrow t = \dfrac{-60 - \sqrt{(60)^2 - 4 * 5 * 60}}{2(-5)} = 6 + \sqrt{24} \approx 10.8$ seconds
(The positive square root gives $t < 6$, so is excluded for this part of the calculation.)