Question

(∗) A wooden post is excavated from some ruins in the centre of Southampton and the University is asked to determine it's probable age. It is known that all living matter has a certain fraction of its carbon as Carbon 14 and the remaining fraction as Carbon 12. Once the matter has died the Carbon 14 decays radioactively to Carbon 12 at a rate proportional to the concentration of Carbon 14 and such that exactly half the Carbon 14 decays to Carbon 12 in 5568 years (called the half-life). Write down the ODE for the concentration of Carbon 14 in dead wood and determine the constants in this equation. Measurements on the post show that it has lost 22% of the amount of Carbon 14 that the living post would have. How old is the post?

Answer

Concentration of Carbon 14 $= C$, the decay is generated by

$\frac{dC}{dt} = -kC$, and the solution is $C = Ae^{-kt}$.

If $C = C_0$ at $\mathrm{t} = 0$ then $\mathrm{C} = \frac{\mathrm{C}_0}{2}$ at $\mathrm{t} = 5568$ years

$\Rightarrow A = C_0 \Rightarrow \frac{C_0}{2} = C_0 e^{-k(5568)}$

$\Rightarrow k = \frac{1}{5568} \ln 2 \approx 0.0001245$

For the age of the wooden post we need to find T where

$C = C_0$ at $\mathrm{t} = \mathrm{T}$, and $\mathrm{C} = (1 - 0.22)\mathrm{C}_0$ at $\mathrm{t} = 2000$.

So $C = Ae^{-\left(\frac{1}{5568} \ln 2\right)t}$

and $C_0 = Ae^{-\left(\frac{1}{5568} \ln 2\right)T}$ and $0.78C_0 = Ae^{-\left(\frac{1}{5568} \ln 2\right)(2000)}$

eliminating C_0 and $A \Rightarrow 0.78 = e^{\left(\frac{1}{5568} \ln 2\right)(T-2000)}$

$T = 2000 + 5568 \frac{\ln 0.78}{\ln 2} \approx 4$ AD.