QUESTION

Evaluate (the complex integral) $\int_1^2 z^k dz$, where $k > -1$ is an integer. Explain why it makes sense to evaluate such an integral. What happens if $k = -1$? What happens if $k < -1$?

ANSWER

It makes sense to evaluate $\int_1^2 z^k dz$ as long as the integral is independent of the path. This is the case if $\int_\gamma z^k dz = 0$ around any closed path γ. This is true if z^k has an antiderivative which is the case if $k \neq -1$. (If $k = -1$ then we are not allowed to use Log$z$ as an antiderivative as Log is not analytic in a neighbourhood of 0.) If $k \neq -1$ then $\int_1^2 z^k dz = \frac{1}{k+1}[z^{k+1}]_1^2 = \frac{2^{k+1}-1}{k+1}$.