QUESTION

Let C_R denote the upper half of the circle $|z| = R$, $(R > 2)$, taken in the counterclockwise direction. Show that

$$|\int_{C_R} \frac{2z^2-1}{z^4+5z^2+4}dz| \leq \frac{\pi R(2R^2+1)}{(R^2-1)(R^2-4)}.$$

Then, by dividing the numerator and denominator of the expression on the right by R^4, show that the value of the integral tends to zero as R tends to infinity.

ANSWER

Length of contour is πR. Also,

$$|\frac{2z^2-1}{(z^2+1)(z^2+4)}| \leq |\frac{2|z|^2|+1}{(|z|^2-1)(|z|^2-4)}| = |\frac{2R^2+1}{(R^2-1)(R^2-4)}|$$

Now just apply the Estimation Theorem to get the result.