QUESTION A random sample of size n is taken without replacement from a very large sample of components and r of the sample are found to be defective. Write down an approximate99% confidence interval for the proportion of the population which are defective stating clearly three reasons who your interval is only approximate.

If $n = 400$ show that the length of the longest such interval is about 0.13.

ANSWER 99% CI approximately $\frac{r}{n} \pm 2.58\sqrt{\frac{\frac{r}{n}(1-\frac{r}{n})}{n}}$
The distribution is really Hypergeometric but the batch is very large so the approximate distribution is Binomial n,p, n large hence we can use the normal to approximate. Variance $=p\frac{q}{n}$ but we use $\frac{r}{n}$ for p as an approximation. n=400, Length of interval $2 \times 2.58\sqrt{\frac{\frac{r}{n}(1-\frac{r}{n})}{n}}$. (p(1-p)is maximum when p= $\frac{1}{2}$)
Hence maximum length $2 \times 2.58\sqrt{\frac{\frac{1}{2}\frac{1}{2}}{400}} = \frac{2.58}{20} \approx 0.13$.