

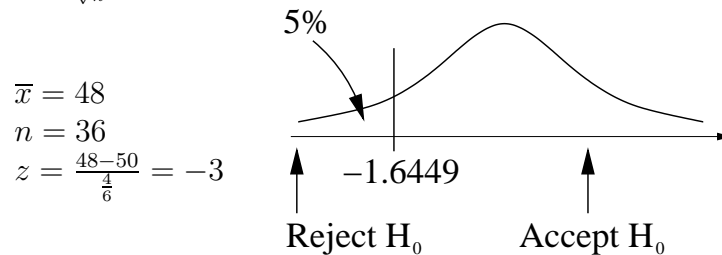
QUESTION The times to carry out a certain job in an industrial process is known to have mean 50 minutes and a standard deviation of 4 minutes. It is claimed that a new method of doing the job will save time. A sample of 36 tests carried out using this new method gave a mean time of 48 minutes. Assuming the same standard deviation for the new method, examine the claim at the 5% level. If the new mean was in fact 48 minutes find the probability of type II error using the standard test.

ANSWER

$$H_0 : \mu = 50 \quad H_1 : \mu < 50 \quad \alpha = 5\%$$

$\sigma = 4$ given, test single mean 1a.

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$$



Hence reject H_0 , accept $H_1 : \mu < 50$.

If $\mu_{\text{new}} = 48$ $\bar{x} \sim N(48, \frac{4^2}{36})$

$$\begin{aligned}
 P(\text{Type II error}) &= P(\text{accept } h_0 | H_1 \text{ true}) \\
 &= P(Z > -1.6449) \\
 &= P\left(\frac{\bar{x} - 50}{6} > -1.6449\right) \\
 &= P(\bar{x} > 48.9034) \\
 &= 1 - \Phi\left(\frac{48.9034 - 48}{\frac{4}{6}}\right) \\
 &= 1 - \Phi(1.3551) \approx 1 - 0.912 \approx 0.088
 \end{aligned}$$