QUESTION The life-time of a type of electric bulb can be assumed to have an exponential distribution with mean $\frac{1}{\lambda}$. 8 such bulbs are tested.

In each of the cases below write down the likelihood function and hence estimate λ by the method of maximum likelihood.

(i) the bulbs fail at times 1.1, 1.3, 1.5, 1.7, 1.9, 2.1, 2.3, 2.5 thousand hours;

(ii) the first five bulbs to fail do so at times 1.1, 1.3, 1.5, 1.7, 1.9 thousand hours, the remainder are still working after 2.0 thousand hours;

(iii) three bulbs fail between 1.0 and 1.6 thousand hours, three between 1.6 and t.t thousand hours and two between 2.2 and 2.8 thousand hours.

ANSWER

(i) $L(\lambda) = \lambda e^{-1.1\lambda}\lambda e^{-1.3\lambda}\lambda e^{-1.5\lambda}\lambda e^{-1.7\lambda}\lambda e^{-1.9\lambda}e^{-2.1\lambda}\lambda e^{-2.3\lambda}\lambda e^{-2.5\lambda} = \lambda^8 e^{14.4\lambda}$

$\ln\lambda = 8\ln\lambda - 14.4\lambda$

$\frac{\partial \ln L(\lambda)}{\partial\lambda} = \frac{8}{\lambda} - 14.4$ therefore $\hat{\lambda} = \frac{8}{14.4} = 0.556$

(ii) $L(\lambda) = \lambda e^{-1.1\lambda}\lambda e^{-1.3\lambda}\lambda e^{-1.5\lambda}\lambda e^{-1.7\lambda}\lambda e^{-1.9\lambda}(e^{-2.0\lambda})^3 = \lambda^5 e^{13.5\lambda}$

from above $\hat{\lambda} = \frac{5}{13,5} = 0.370$

(iii) $L(\lambda) = (e^{-\lambda} - e^{1.6\lambda})^3(e^{-1.6\lambda} - e^{-2.2\lambda})^3((e^{-2.2\lambda} - e^{-2.8\lambda})^3 = e^{-12.2\lambda}(1 - e^{-0.6\lambda})^8$

$\ln L(\lambda) = -12.2\lambda + 8\ln(1 - e^{-0.6\lambda})$

$\frac{\partial \ln L(\lambda)}{\partial\lambda} = -12.2 + \frac{8\times 0.6e^{-0.6\lambda}}{1-e^{-0.6\lambda}}$

$12.2 = \frac{4.8e^{-0.6\hat{\lambda}}}{1-e^{-0.6\hat{\lambda}}} = 17e^{-0.6\hat{\lambda}}$

$e^{0.6\hat{\lambda}} = \frac{17}{12.2}$ $\hat{\lambda} = 0.553$