Question

Use the given solution to find a second solution for each of the following differential equations:

a)	$x^2y'' + 2xy' - 2y = 0$		$y_1(x) = x$	(*)
b)	$(x-1)y'' - xy' + y = 0$	$x > 1$	$y_1(x) = e^x$	(*)

Answer

Check that $y_1 = x$ is a solution to the equation

This follows since $y_1' = 1$ and $y_1'' = 0$ and putting these into the equation makes it true.

Equation is in standard form with $p(x) = \dfrac{2}{x}$ and $q(x) = \dfrac{-2}{x^2}$. Use method of reduction of order with $y_2 = vy_1$ where $v(x) = \displaystyle\int \left(\frac{1}{y_1^2(x)}\ e^{-\int p(x)\,dx} \right)\, dx$

so that $v(x) = \displaystyle\int \left(\frac{1}{x^2}\ e^{-\int \frac{2}{x}\,dx} \right)\, dx = \int \left(\frac{1}{x^2}\ e^{-2\ln x} \right)\, dx = \int \left(\frac{1}{x^2}\ \frac{1}{x^2} \right)\, dx = \frac{-1}{3x^3}$

Hence the general solution is $y(x) = Ax + \dfrac{B}{x^2}$