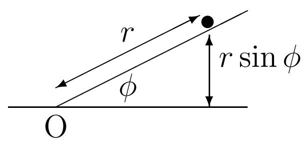
Question

A particle of mass m rests on a smooth plane. The plane is raised to an incline ϕ at a constant rate α ($\phi = 0$ at $t = 0$), causing the particle to move down the plane. Express the Lagrangain in polar coordinates of the particle in a coordinate system whose origin is at the foot of the plane. Hence determine the motion of the particle.

Answer



$\phi = \alpha t$

$$L = K.E. - P.E. = \frac{1}{2}m(\dot{r}^2 - r^2\dot{\phi}^2) - mgr\sin\alpha t$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0 \Rightarrow \ddot{r} - \alpha^2 r = -g\sin\alpha t$$

This has solution $r(t) = Ae^{\alpha t} + Be^{-\alpha t} + \dfrac{g}{2\alpha^2}\sin\alpha t$

Initially $r(0) = r_0$, $\dot{r}(0) = 0$ whence we can find A and B

Thus $r(t) = \dfrac{1}{2}\left[r_0 - \dfrac{g}{2\alpha^2}\right]e^{\alpha t} + \dfrac{1}{2}\left[r_0 + \dfrac{g}{2\alpha^2}\right]e^{-\alpha t} + \dfrac{g}{2\alpha^2}\sin\alpha t$