Question

If 16 digits are chosen from a table of random digits, what is the probability that their average will lie between 4 and 6?

Answer

Here $n = 16$

Let X_i be the chosen digit in the ith draw.

$X_i = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ with probability $\frac{1}{10}$

Therefore $E(X_i) = \mu = \frac{0 + 1 + 2 + ... + 9}{10} = \frac{9 \times 10}{2 \times 10} = 4.5$

$E(X_i^2) = \frac{0^2 + 1^2 + 2^2 + ... + 9^2}{10} = \frac{9(10)(19)}{6 \times 10} = \frac{57}{2} = 28.5$

[Remember $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$]

Now $\sigma^2 = E(X_i^2) - \{E(X_i)\}^2 = 28.5 - (4.5)^2 = 8.25$

We want $P(4 < \bar{X}_n < 6)$ where $\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} \sim N(0, 1)$ approximately.

$$
\begin{aligned}
&= P\left\{\frac{\sqrt{16}}{\sqrt{8.25}}(4 - 4.5) < \frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} < \frac{\sqrt{16}}{\sqrt{8.25}}(6 - 4.5)\right\} \\
&= P\{-0.69 < Z < 2.09\} \\
&= 0.7366
\end{aligned}
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