Question

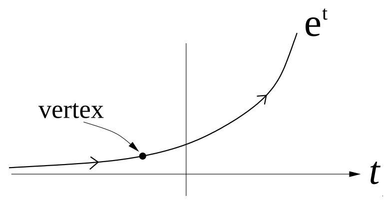
Sketch each of the plane curves γ given by the following parametrizations:

(i)	$\gamma(t) = (t, e^t)$	**(iv)**	$\gamma(t) = (t - \sin t, 1 + \cos t)$
(ii)	$\gamma(t) = (t^3, t^4)$	**(v)**	$\gamma(t) = (e^{kt}\cos t, e^{kt}\sin t) \quad (k \neq 0)$
(iii)	$\gamma(t) = (t - t^2, t + t^2)$	**(vi)**	$\gamma(t) = (t^2 - 1, t^3 - t)$

Answer

(i)

$$\begin{aligned} K(t) &= e^t(1+e^{2t})^{-\frac{3}{2}} > 0 \\ K'(t) &= e^t(1+2^t)^{-\frac{5}{2}}.(1-2e^{2t}) \end{aligned}$$

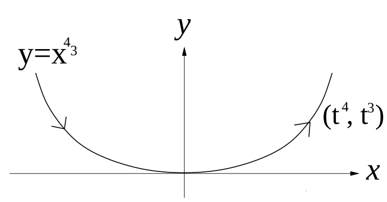


So $K' = 0$ where $e^{2t} = \frac{1}{2}$.

(ii) $K(t) = 12t^-(9+16t^2)^{-\frac{3}{2}}$**, defined for all $t \neq 0$. (Note also $\gamma'(0) = 0$).**

$$\left.\begin{array}{llll} |K| & \to \infty & \textbf{as} & t \to 0 \\ & \to 0 & \textbf{as} & t \to \infty \end{array}\right\}$$

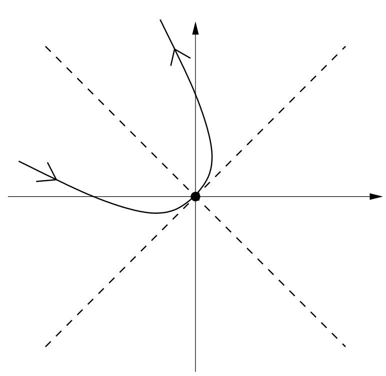
No vertices



At origin, deceptive@ $\gamma(t)$ slows to instantaneous halt.

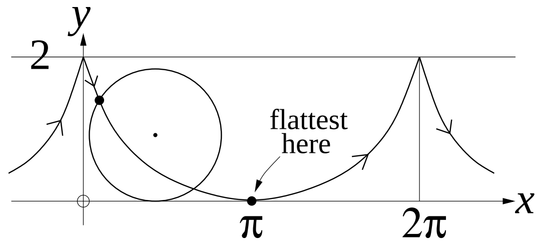
$\frac{dy}{dx}$ **exists here $(=0)$, but $\frac{d^2y}{dx^2}$ doesn't.**

(iii) $x + y = 2t$, $-x + y = 2t^2$**, so rotation by 45 degrees shows curve is a parabola.**



$K(t) = 4(2+8t^2)^{-\frac{3}{2}}$, **max when** $t=0$.

(iv) This is a cycloid: take standard cycloid (for circle of radius 1), reflect in the x**-axis and translate by** 2 **in the** y**-direction.**



$\gamma'(t) = (1-\cos t, -\sin t)$, **which is zero when** $t = 2n\pi$ $(n = 0, \pm 1, \pm 2, \cdots)$.

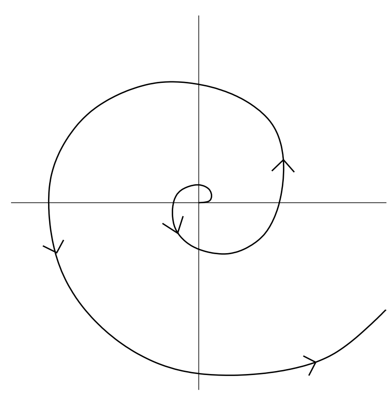
$$\begin{aligned} K(t) &= -\frac{(1-c).c + ss}{((1-c)^2+s^2)^{3/2}} \\ &= \frac{1-c}{(2-2c)^{\frac{3}{2}}} \\ &= 2^{-\frac{3}{2}}(1-c)^{-\frac{1}{2}} \\ &\quad \textbf{where } c = \cos t \ (\neq 1) \\ &\quad \textbf{and } s = \sin t \\ &= 1/4 \sin\frac{t}{2} \end{aligned}$$

defined except when $t = 2n\pi$, $(n \in \mathbf{Z})$.

So $k > 0$, **and as** t **goes from** 0 **to** 2π **we see** K **increases to** $\frac{1}{4}$ **(at** $t=\pi$**), then increases again.**

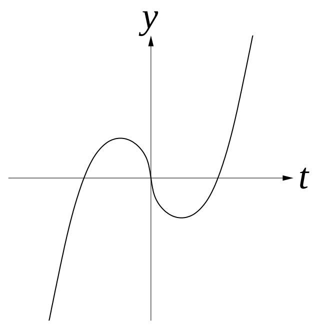
(v) $K(t) = e^{-kt}(1+K^2)^{-\frac{1}{2}}$.

$\to 0$ **as** $t \to +\infty$
$\to 0\infty$ **as** $t \to -\infty$, **i.e. as** $\gamma(t) \to (0,0)$

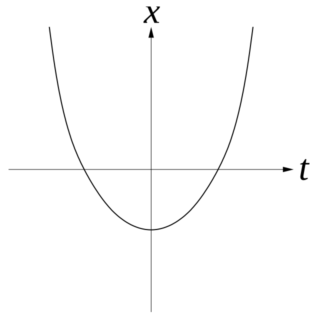


Spiral - radius increases exponentially.

(vi)

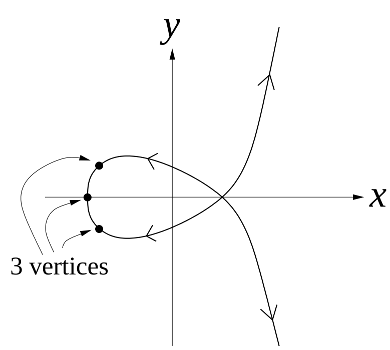


$y(t)$ **goes**



While $x(t)$ **goes**

If $t \to -t$**, then** $x \to x$ **and** $y \to -y$**.**



$K(t) = (6t^2 + 2)(9t^4 - 2t^2 + 1)^{-\frac{3}{2}}, > 0$

defined for all $t \in \Re$ **since** $9u^2 - 2u + 1$ **has no real roots.**

$K'(t) = -24t(9t^4 + 4t^2 - 1)(9t^4 - 2t^2 + 1)^{-\frac{5}{2}}, = 0$ **when** $t = 0$ **or** $t^2 = \frac{1}{9}(-2 + \frac{13}{)}$**.**