Question

Two tanks both initially contain 200 L of fresh water. Starting at $t = 0$ brine containing 5 kg/L of salt is added to the first tank at the rate of 2 L/min. This first tank is continually stirred. The uniform solution from the first tank is transferred to the second tank at the rate of 2 L/min. This second tank is also stirred. A uniform mixture leaves the second tank also at a rate of 2 L/min. What is the concentration of the mixture leaving the first tank at time t? What is the concentration of the mixture leaving the second tank at time t?

Answer

$x(t) =$ salt in tank 1 $\qquad x(0) = 0$
$y(t) =$ salt in tank 2 $\qquad y(0) = 0$
The water balance is such that both tanks contain 200 litres at all times.
Salt balance in tank 1:

$$\begin{array}{cccc} \textit{rate of change of salt} & = \textit{rate of salt in} & - & \textit{rate of salt out} \\ \frac{dx}{dt} & = 5 * 2 & - & \frac{x}{200} * 2 \\ \frac{dx}{dt} & = 10 & - & \frac{x}{100} \end{array} \quad (1)$$

Salt balance in tank 2:

$$\begin{array}{cccc} \textit{rate of change of salt} & = \textit{rate of salt in} & - & \textit{rate of salt out} \\ \frac{dy}{dt} & = \frac{x}{200} * 2 & - & \frac{y}{200} * 2 \\ \frac{dy}{dt} & = \frac{x}{100} & - & \frac{y}{100} \end{array} \quad (2)$$

Solve (1) as a linear equation with $x(0) = 0$, and we get

$$x(t) = 1000\left(1 - e^{-\frac{t}{100}}\right)$$

Put this solution into (2) and solve as a linear equation with $y(0) = 0$, and we get

$$y(t) = 1000\left(1 - e^{-\frac{t}{100}} - \frac{te^{-\frac{t}{100}}}{1000}\right)$$

So the concentration is $\dfrac{x}{100}$ and $\dfrac{y}{100}$.