Question
(∗) Assume that the population of the earth changes at a rate proportional to the current population.

1. Write the ODE satisfied by the population
2. In 1650 the population is estimated to have been about 600 million, and in 1950 about 2,800 million. By solving the ODE and fitting this data estimate the population at time t (years AD).
3. Using this fitted solution, and assuming that the greatest population the earth can support is 2.5×10^{10} people, in what year will this limit be reached?

Answer
$N(t)$ =population

a) $\dfrac{dN}{dt} = kN$

b) $t = 1650, \quad N = 600 * 10^6$

$t = 1950, \quad N = 2800 * 10^6$

from a) $N = Ae^{kt}$

$$\begin{aligned} N(1650) &\Rightarrow 600 * 10^6 = Ae^{k1650} \Rightarrow A = 6 * 10^8 e^{-k1650} \\ N(1950) &\Rightarrow 2800 * 10^6 = Ae^{k1950} = 6 * 10^8 e^{k(1950-1650)} \\ &\Rightarrow 2800 * 10^6 = 6 * 10^8 e^{300k} \end{aligned}$$

$$k = \frac{1}{300} \ln\left(\frac{14}{3}\right) \approx 0.0015 \text{ per year.}$$

c) $2.5 * 10^{10} = 6 * 10^8 e^{k(t-1650)}$

$$\Rightarrow \frac{\ln\left(\frac{2.5*10^2}{6}\right)}{k} + 1650 = t$$

$$726 + 1650 \approx t$$

$$2376 \approx t$$