Question

Let A be the Euclidean circle in **H** with Euclidean centre $1+2i$ and Euclidean radius 1. We know from class that A is also a hyperbolic circle. Determine its hyperbolic centre and hyperbolic radius.

Answer

The hyperbolic center is the midpoint of the hyperbolic line segment joining two points on a hyperbolic diameter, such as $1 + i$ and $1 + 3i$. Since $d_{\mathbf{H}}(1 + i, 1 + 3i) = \ln(3)$, the hyperbolic center is the point $1 + ai$ which satisfies

$$d_{\mathbf{H}}(1 + i, 1 + ai) = \frac{1}{2}\ln(3) = d_{\mathbf{H}}(1 + ai, 1 + 3i)$$

$$\ln(a) = ds\frac{1}{2}\ln(3) = \ln\left(\frac{3}{a}\right) = \ln(3) - \ln(a)$$

So, $a = \sqrt{3}$ and so the hyperbolic center is $\underline{1 + \sqrt{3}i}$.

The hyperbolic radius is $d_{\mathbf{H}}(1 + i, 1 + \sqrt{3}i) = \frac{1}{2}\ln(3)$.