Question
Determine whether the Möbius transformation

$$m(z) = \frac{4z - 5}{2z + 3}$$

is parabolic,elliptic or loxodromic, and determine it's fixed points.If it is elliptic or loxodromic, determine its multiplier.

Answer
Use classification by trace squared: first normalize so that determinant is 1.
$\det(m) = 12 + 10 == 22$, so m normalized is

$$m(z) = \frac{\frac{4}{\sqrt{22}}z - \frac{5}{\sqrt{22}}}{\frac{2}{\sqrt{22}}z + \frac{3}{\sqrt{22}}}$$

$\text{Trace}^2(m) = \left(\frac{4}{\sqrt{22}} + \frac{3}{\sqrt{22}}\right)^2 = \frac{49}{22}$
since $0 \leq \text{trace }^2(m) < 4$, m is elliptic.
Find the multiplier from the trace squared.
$(\lambda + \lambda^{-1})^2 = \text{trace}^2(m) = \frac{49}{22}$, where the multiplier of m is λ^2.
Then, $\lambda^2 + \lambda^{-1} + 2 = \frac{49}{22}$, so $\lambda^4 + 1 - \frac{5}{22}\lambda^2 = 0$
So, $\lambda^2 = \frac{[\frac{5}{22} + \sqrt{(\frac{5}{22})^2 - 4}]}{2}$ (and $|\lambda^2| = 1$).
$\frac{5}{44} \pm \frac{\sqrt{1911}i}{44}$

The fixed points of m are the solutions to $m(z) = z$, namely
$4z - 5 = z(2z + 3)$, on $2z^2 - z + 5 = 0$, so
$z = \frac{1}{4}[1 \pm \sqrt{1 - 40}] = \underline{\frac{1}{4}(1 \pm \sqrt{39}i)}$.