Question
Let X have pdf $f(x) = 42x^5(1-x), \quad 0 < x < 1$. Find the pdf of $Y = X^3$. Show that the pdf integrates to 1.

Answer
The range of y is $0 < y < 1$.
Also $x = y^{\frac{1}{3}}$
Therefore $\dfrac{dx}{dy} = \dfrac{1}{3}y^{-\frac{2}{3}}$
Therefore the pdf of Y is

$$\begin{aligned} g(y) &= 42y^{\frac{5}{3}}\left(1-y^{\frac{1}{3}}\right)\cdot\left|\frac{1}{3}y^{-\frac{2}{3}}\right|, \quad 0<y<1. \\ &= 14y\left(1-y^{\frac{1}{3}}\right), \quad 0<y<1 \end{aligned}$$

$$\begin{aligned} \int_0^1 g(y)\,dy &= 14\int_0^1 \left(y-y^{\frac{4}{3}}\right)\,dy \\ &= 14\left(\frac{1}{2}-\frac{1}{7/3}\right) \\ &= 14\left(\frac{1}{2}-\frac{3}{7}\right) \\ &= 14\cdot\frac{7-6}{14} \\ &= 1 \end{aligned}$$