

Question

Suppose that X is uniformly distributed over the interval $(0,1)$, i.e. the pdf of X is

$$f(x) = \begin{cases} 1, & \text{for } 0 < x < 1; \\ 0, & \text{otherwise} \end{cases}$$

Find the pdf of $Y = -2 \log(X)$.

Answer

$$y = -2 \log x \Rightarrow x = e^{-\frac{y}{2}}$$

The transformation is decreasing.

The range of y is $0 < y < \infty$.

$$\frac{dx}{dy} = e^{-\frac{y}{2}} \left(\frac{1}{-2} \right)$$

The pdf of Y is $g(y) = 1 \left| \frac{dx}{dy} \right| = \frac{1}{2} e^{-\frac{y}{2}}, \quad 0 < y < \infty$.

The distribution is called the χ^2 distribution with 2 degrees of freedom.