Question

Use L'Hopital rule to evaluate the following limits

(a) $\lim\limits_{x\to 1} \dfrac{x^2 - 2x + 1}{x^2 - 3x + 2}$

(b) $\lim\limits_{x\to 0} \dfrac{\cos x - \cos 2x}{x^2}$

(c) $\lim\limits_{x\to 1} \dfrac{\sin \pi x}{\cos \frac{\pi x}{2}}$

Answer

(a) Consider the expression $\dfrac{x^2 - 2x + 1}{x^2 - 3x + 2}$. Both numerator and denominator tend to zero as $x \to 1$.

Using L'Hopital's rule gives $\lim\limits_{x\to 1} \dfrac{x^2 - 2x + 1}{x^2 - 3x + 2} = \lim\limits_{x\to 1} \dfrac{2x - 2}{2x - 3} = \dfrac{0}{-1} = 0$

(b) Consider the expression $\dfrac{\cos x - \cos 2x}{x^2}$. Both numerator and denominator tend to zero as $x \to 0$.

Using L'Hopital's rule gives

$$\lim_{x\to 0} \frac{\cos x - \cos 2x}{x^2} = \lim_{x\to 0} \frac{\frac{d}{dx} \cos x - \cos 2x)}{\frac{d}{dx}(x^2)} = \lim_{x\to 0} \frac{-\sin x + 2\sin 2x}{2x}$$

Again numerator and denominator both tend to zero as $x \to 0$. We use L'Hopital's rule again to give

$$\lim_{x\to 0} \frac{-\sin x + 2\sin 2x}{2x} = \lim_{x\to 0} \frac{-\cos x + 4\cos 2x}{2} = \frac{3}{2}$$

(c) Consider the expression $\dfrac{\sin \pi x}{\cos \frac{\pi x}{2}}$. Both numerator and denominator tend to zero as $x \to 1$.

Using L'Hopital's rule gives

$$\lim_{x\to 1} \frac{\sin \pi x}{\cos \frac{\pi x}{2}} = \frac{\pi \cos \pi x}{-\frac{1}{2}\pi \sin \frac{\pi x}{2}} = \frac{\pi \cos \pi}{-\frac{1}{2}\pi \sin \frac{\pi}{2}} = \frac{-\pi}{-\frac{1}{2}\pi} = 2$$