Question

Solve the equations

(i) $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$

(ii) $4\dfrac{d^2y}{dx^2} - 12\dfrac{dy}{dx} + 9y = 0$

(iii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = 0$

Answer

(i) $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$

Try solution

$$\left\{ \begin{array}{ccc} y & = & Ae^{kx} \\ \dfrac{dy}{dx} & = & Ake^{kx} \\ \dfrac{d^2y}{dx^2} & = & Ak^2e^{kx} \end{array} \right.$$

$\Rightarrow$ auxiliary equation

$k^2 - 5k + 6 = 0$

$\Rightarrow (k-2)(k-3) = 0$

$\Rightarrow k = 2$ or $k = 3$

Unequal real roots. Thus the general solution is (see notes)

$$\underline{y = Ae^{2x} + Be^{3x}}$$

$A,\ B$ arbitrary constants

(ii) $4\dfrac{d^2y}{dx^2} - 12\dfrac{dy}{dx} + 9y = 0$

Try solution

$$\begin{cases} y &= Ae^{kx} \\ \dfrac{dy}{dx} &= Ake^{kx} \\ \dfrac{d^2y}{dx^2} &= Ak^2e^{kx} \end{cases}$$

$\Rightarrow$ auxiliary equation

$4k^2 - 12k + 9 = 0$

$\Rightarrow (2k-3)^2 = 0$

$\Rightarrow k = \dfrac{3}{2},\ \dfrac{3}{2}$

Two equal real roots. Thus the general solution is (see notes)

$$\underline{y = (A + Bx)e^{\frac{3x}{2}}}$$

(iii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = 0$

Try solution

$$\begin{cases} y &= Ae^{kx} \\ \dfrac{dy}{dx} &= Ake^{kx} \\ \dfrac{d^2y}{dx^2} &= Ak^2e^{kx} \end{cases}$$

$\Rightarrow$ auxiliary equation is

$k^2 + 2k + 2 = 0$

$\Rightarrow k = \dfrac{-2 \pm \sqrt{4 - 4\cdot 2}}{2} = \dfrac{-2 \pm \sqrt{-4}}{2} = -1 \pm i$

This is an imaginary roots case.

Thus $y = Ae^{(-1+i)}x + Be^{(-1-i)}x = \underline{e^{-x}(Ae^{ix} + Be^{-ix})}$

Alternatively $\underline{y = e^{-x}(C\cos x + D\sin x)}$

Probably better since this is obviously real, like the equation.