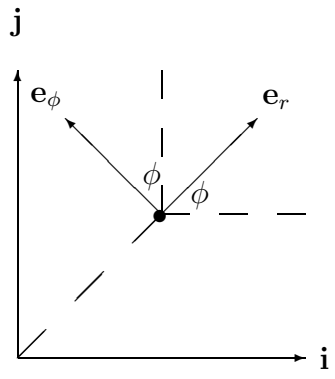
Question

Express the basis vectors $\mathbf{e}_r$ and $\mathbf{e}_\phi$ of polar coordinates the time derivative of the basis vectors of Cartesian coordinates $\mathbf{i}$ and $\mathbf{j}$. Suppose that $\mathbf{e}_r$ and $\mathbf{e}_\phi$ depend on time. Show that

(a) $\dot{\mathbf{e}}_r = \dot{\phi}\mathbf{e}_\phi$

(b) $\dot{\mathbf{e}}_\phi = -\dot{\phi}\mathbf{e}_r$

Answer



$$\begin{aligned} \mathbf{e}_r &= \mathbf{i}\cos\phi + \mathbf{j}\sin\phi \\ \mathbf{e}_\phi &= -\mathbf{i}\cos\left(\frac{\pi}{2} - \phi\right) + \mathbf{j}\cos\phi \\ &= -\mathbf{i}\sin\phi + \mathbf{j}\cos\phi \end{aligned}$$

$$\dot{\mathbf{e}}_r = \frac{d}{dt}(\mathbf{i}\cos\phi + \mathbf{j}\sin\phi) = \dot{\phi}(-\mathbf{i}\sin\phi + \mathbf{j}\cos\phi) = \dot{\phi}\mathbf{e}_\phi$$

$$\dot{\mathbf{e}}_\phi = \frac{d}{dt}(-\mathbf{i}\sin\phi + \mathbf{j}\cos\phi) = \dot{\phi}(-\mathbf{i}\cos\phi - \mathbf{j}\sin\phi) = -\dot{\phi}\mathbf{e}_r$$