

Question

Discuss Jordan measurability and content of sets of the following types

- i) a countable set with one point of accumulation.
- ii) a countable set with 10^6 points of accumulation.

Answer

Firstly we assume that S is bounded.

- i) Cover the point of accumulation with a rectangle R_1 such that $|R_1| < \frac{\epsilon}{2}$. There are a finite number of points left over. Cover these by rectangles R_2, \dots, R_n such that $|R_i| < \frac{\epsilon}{2^i}$ then $\sum_{i=1}^n |R_i| < \epsilon$. So $C^*(S) = 0$ and $c_*(S) = 0$ also.
- ii) Cover the 10^6 points of accumulation by the rectangles R_1, \dots, R_{10^6} such that $|R_i| < \frac{\epsilon}{2^i}$. There are a finite number of points left over. Cover these by rectangles R_{10^6}, \dots, R_n , $|R_i| < \frac{\epsilon}{2^i}$. Again $\sum_{i=1}^n |R_i| < \epsilon$. So $C^*(S) = 0$ and also $c_*(S) = 0$.