Question
Deduce as many of the following properties as you can from the additive law

i) $m(\phi) = 0$

ii) $m(A - B) = m(A) - m(A \cap B)$

iii) $B \subseteq A \Rightarrow m(A - B) = m(A) - m(B)$

iv) $m(A \cup B) = m(A) + m(B) - m(A \cap B)$

v) $m(A) \geq 0$

vi) $A \subseteq B \Rightarrow m(A) \leq m(B)$

Answer
The additive law says $A \cap B = \phi \Rightarrow m(A \cup B) = m(A) + m(B)$

i) Put $A = B = \phi$, then $A \cap B = \phi$, $A \cup B = \phi$.
$m(\phi) = m(\phi) + m(\phi)$, therefore $m(\phi) = 0$

ii) $A = (A \cap B) \cup (A - B)$ $\quad$ $(A \cap B) \cap (A - B) = \phi$
Therefore $m(A) = m(A \cap B) + m(A - B)$
Therefore $m(A - B) = m(A) - m(A \cap B)$

iii) $B \subseteq A \Rightarrow A \cap B = B$
Therefore $m(A - B) = m(A) - m(B)$

iv) $A \cup B = A \cup (B - A)$ $\quad$ $A \cap (B - A) = \phi$
Therefore $m(A \cup B) = m(A) + m(B - A)$
$= m(A) + m(B) - m(A \cap B)$

vi) $A \subseteq B \Rightarrow m(B) = m(A) + m(B - A)$

v) We cannot prove $m(A) \geq 0$, as the following example shows.
$S = \{a, b\}$, $m(\phi) = 0$ $m(\{a\}) = 1$ $m(\{b\}) = -1$
$m(S) = 0$
Then for all $A, B \subset S$ $\quad$ $A \cap B = \phi \Rightarrow m(A \cup B) = m(A) + m(B)$
But $m(A) \ngeq 0$ for all A.